Dynamics 4

Learning Module 6

Transverse vibration and flexural waves on beams

THE UNIVERSITY *of* EDINBURGH
School of Engineering

DYNAMICS 4

LM6 GUIDE 26 27

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Learning Module 6 | Transverse vibration and flexural waves on beams

Estimated time: 15 hours Complete module before: 25 November

1. Start by reading the Course Notes: For this module you should read **Section 3.3.** Take hand-written notes as you read.

2. Attend the lectures and tutorial sessions and, if you need, watch the lecture recordings later. This will help consolidate what you learned. Keep taking notes.

3. Attempt to solve **Questions 6.1 to 6.8** in **Exercise Sheet 6**. Compare your solutions to the numerical solutions provided at the end of each question. Write down any questions you might have.

4. Compare your solutions to the worked-out example solutions to **Questions 6.1 to 6.8**. Discuss any questions you might have in the tutorial sessions. Arrange to meet with your lecturer if you need further clarifications.

5. Be active in the Learn discussion board thread for this Learning Module, which is labeled **transverse_waves**. You can post your own questions and help clarify or answer the questions of others.

6. Have a look at the additional resources provided in the **Resources folder** for Learning Module 6, on Learn. You should also read **Section 3.4** of the Course Notes with examples of real life applications.



In this module

Dynamics of continuous systems
Longitudinal waves in solids
Travelling and standing wave solutions
Boundary conditions

Learning outcomes

Derive equations of motion for dynamic systems from energy-based approaches (e.g. Lagrange's method)

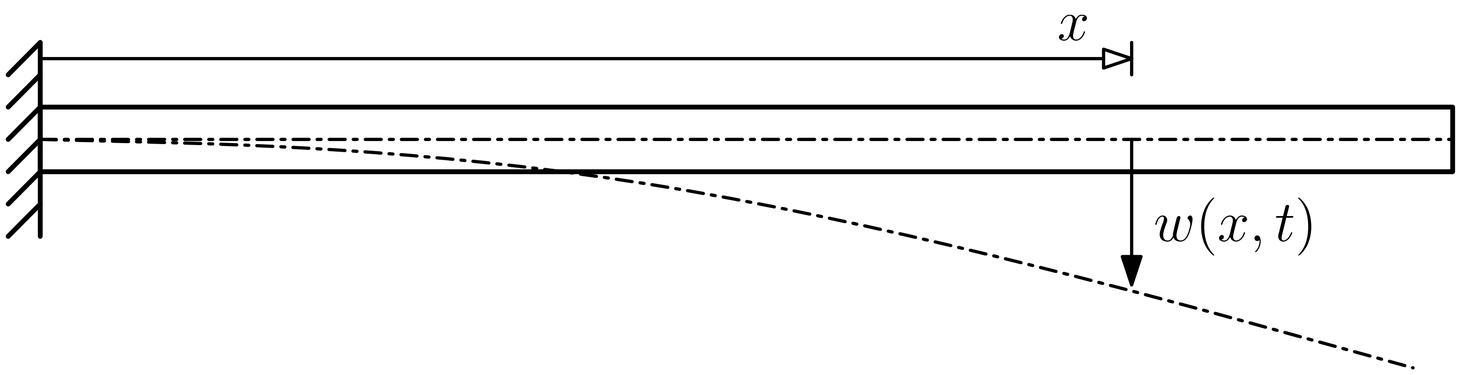
Know the common wave equations for basic structural elements and be able to use these to better understand the response of simple structural elements

This is a study and learning guide to Learning Module 6. You can find the corresponding lecture recordings and slides on Learn. This module has a dedicated thread in the discussion board on Learn.

Please note that the course time and the recommended *before dates*, are estimated and may differ from person to person. You should not see these as targets but instead as a way to help you plan your semester and the preparation for exams. The estimated time includes going through all the course materials, interaction in the discussion boards and self-study time.

Transverse vibration

- Flexural waves
  - Uniform beams
  - Cross-section remains plane and at right angle to neutral axis
  - Small angles (slope) and small deflections
  - Thin beams ($L/d > 10$)



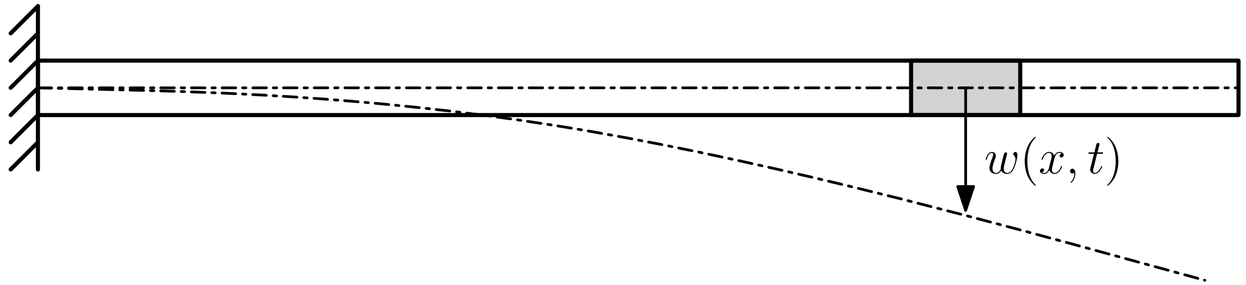
Transverse vibration

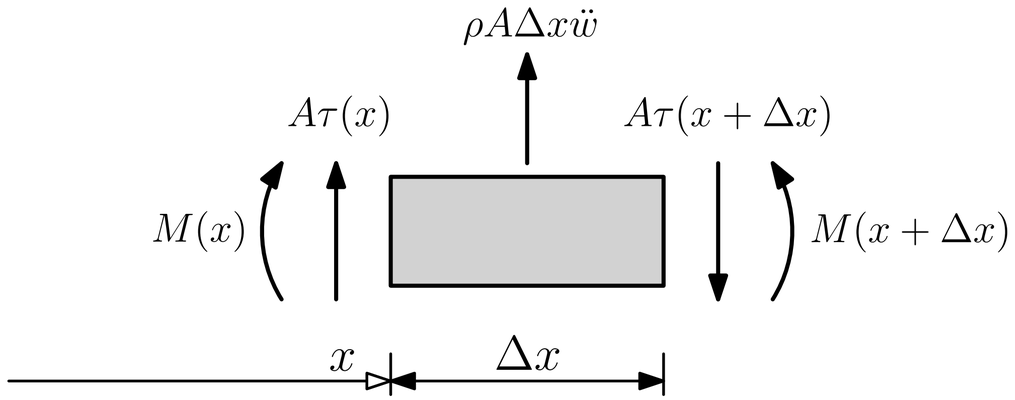
- Euler-Bernoulli equation
  - Bending moment from beam statics

$$EI\frac{\partial^2 w}{\partial x^2} = -M(x,t)$$

Transverse vibration

- Dynamic equilibrium





Transverse vibration

- Dynamic equilibrium (balance of moments):

$$M(x) + A\tau(x)\frac{\Delta x}{2} + A\left[\tau(x) + \frac{\partial \tau}{\partial x}\Delta x\right]\frac{\Delta x}{2} - \left[M(x) + \frac{\partial M}{\partial x}\Delta x\right] = 0$$

- Shear force on cross-section:

$$A\tau(x) = \frac{\partial M}{\partial x} = -EI\frac{\partial^3 w}{\partial x^3}$$

Transverse vibration

- Dynamic equilibrium (balance of forces):

$$\rho A \Delta x \frac{\partial^2 w}{\partial t^2} + A\left[\tau(x) - \tau(x + \Delta x)\right] = 0$$

- Euler-Bernoulli equation for uniform beam:

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = 0$$

Transverse vibration

- Travelling wave solution:

$$w(x,t) = C \sin\left[\frac{2\pi}{\lambda}(x - ct)\right]$$

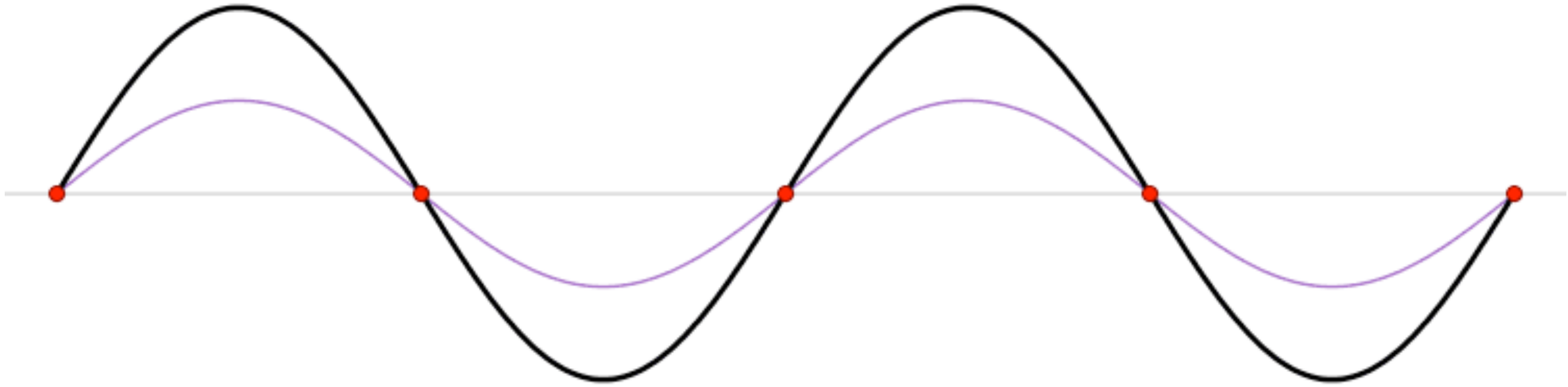
- Inserting into Euler-Bernoulli equation yields:

$$\left(\frac{2\pi}{\lambda}\right)^2 = \frac{\rho A c^2}{EI}$$

Dispersive wave:
Wave speed depends on wavelength

Transverse vibration

- Travelling vs. standing waves



Transverse vibration

- Euler-Bernoulli equation (recall):

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = 0$$

- Standing wave solution:

$$w(x,t) = \varphi(x)\theta(t) = \varphi(x)\cos(\omega t)$$

... yields:

$$\left[EI \frac{\partial^4 \varphi(x)}{\partial x^4} - A\rho\omega^2 \varphi(x) \right] \cos(\omega t) = 0$$

Transverse vibration

- Euler-Bernoulli equation:

$$\left[EI\frac{\partial^4\varphi(x)}{\partial x^4} - A\rho\omega^2\varphi(x)\right]\cos(\omega t) = 0$$

... true for all t only if:

$$\frac{\partial^4\varphi(x)}{\partial x^4} - k^4\varphi(x) = 0 \quad \text{with} \quad k^4 = \frac{A\rho\omega^2}{EI}$$

Transverse vibration

- Solution for shape function:

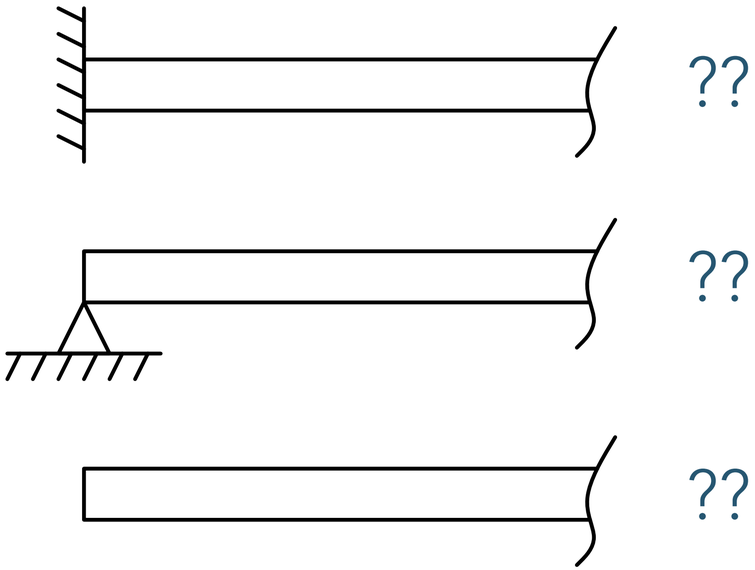
$$\frac{\partial^4 \varphi(x)}{\partial x^4} - k^4 \varphi(x) = 0 \quad \text{with} \quad k^4 = \frac{A\rho\omega^2}{EI}$$

... is the standing flexural wave on a uniform beam:

$$\varphi(x) = C_1 \cos(kx) + C_2 \sin(kx) + C_3 \cosh(kx) + C_4 \sinh(kx)$$

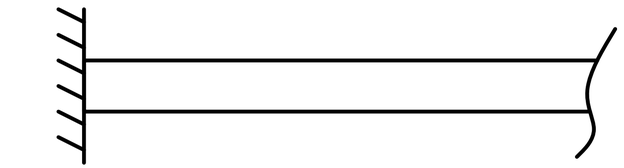
Transverse vibration

- Boundary conditions

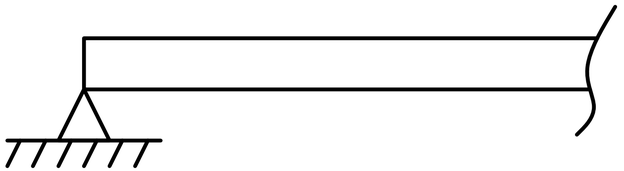


Transverse vibration

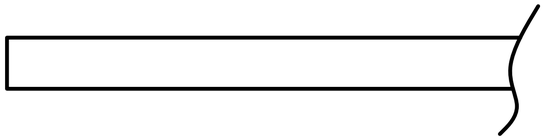
- Boundary conditions: True or False?



$$w(0,t) = 0 \Rightarrow \varphi(0) = 0$$
$$w''(0,t) = 0 \Rightarrow \varphi''(0) = 0$$



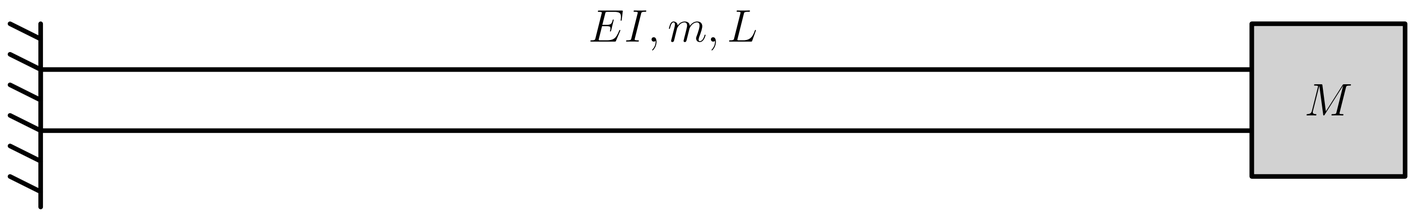
$$w(0,t) = 0 \Rightarrow \varphi(0) = 0$$
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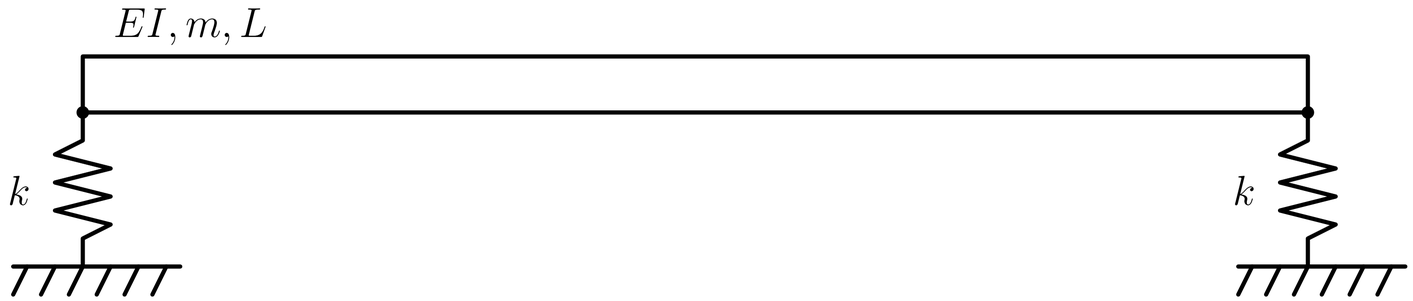


$$w''(0,t) = 0 \Rightarrow \varphi''(0) = 0$$
$$w'''(0,t) = 0 \Rightarrow \varphi'''(0) = 0$$

Transverse vibration

- General boundary conditions





Transverse vibration

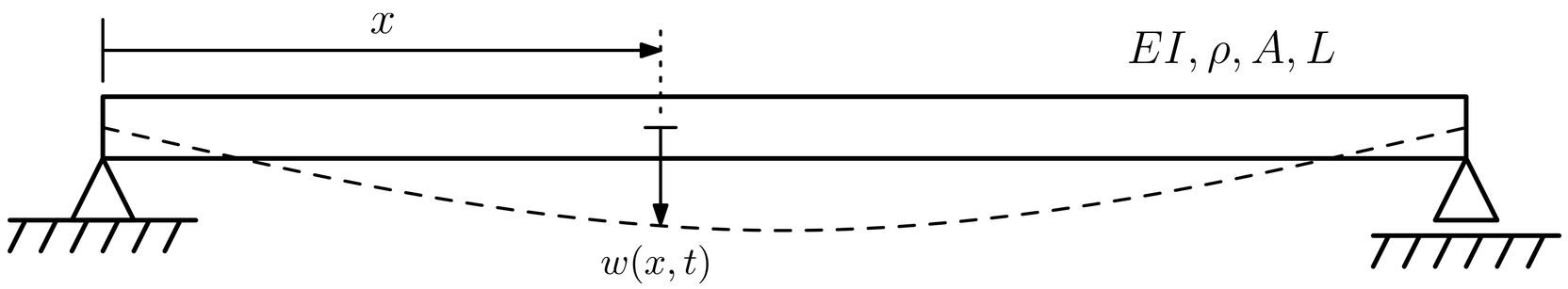
- General boundary conditions
- Force balance: Force on boundary = Shear force

$$F = A\tau(x)|_{x=l} = \frac{\partial M}{\partial x} = -EI\frac{\partial^3 w}{\partial x^3}$$

- Force can be due to:
  - Spring (depends on displacement)
  - Inertia (depends on acceleration)
  - Damping (depends on velocity)

Example (CN)

Determine the flexural vibration modes of a uniform beam simply supported at both ends.



Example (CN)

A steel pipe is 10 m long with diameter 1.2 m and thickness 8 mm. The stiffness of support B is 1.2 MN/m, *E* is 207 GPa and the density is 7840 kg/m^3.

a) Determine the first two frequencies of flexural vibration of the pipe.

b) Determine the frequency considering the pipe to be rigid.

